

NEUTRINO DARK ENERGY

E. I. Guendelman and A. B. Kaganovich

Physics Department, Ben Gurion University of the Negev, Beer Sheva 84105, Israel

E-mail: guendel@bgu.ac.il and alexk@bgu.ac.il

There exist field theory models where the fermionic energy-momentum tensor contains a term proportional to $g_{\mu\nu}\bar{\Psi}\Psi$ which can be responsible for a dark matter to dark energy transmutation. We study some cosmological aspects of the new field theory effect where nonrelativistic neutrinos are obliged to be drawn into cosmological expansion (by means of dynamically changing their own parameters). This becomes possible as the magnitudes of the cold neutrino and vacuum energy densities are comparable. Some of the features of such Cosmo-Low Energy Physics (CLEP) state in the toy model of the late time universe filled with homogeneous scalar field and uniformly distributed nonrelativistic neutrinos: neutrino mass increases as $a^{3/2}$ ($a = a(t)$ is the scale factor); its energy density scales as a sort of dark energy and its equation-of-state approaches $w = -1$ as $a \rightarrow \infty$; the total energy density of such universe is less than it would be in the universe free of fermionic matter at all. CLEP state can be realized in the framework of an alternative gravity and matter fields theory. The latter is reduced to canonical General Relativity when the fermionic matter built of the first two fermion families is only taken into account. In this case also the 5-th force problem is resolved automatically.

PACS: 04.50.+h, 98.80.Cq

Introduction. One of the most fundamental questions facing modern physics is the nature of the dark matter and the dark energy. A particularly puzzling aspect is that at the present time the dark matter and the dark energy densities appear to be of the same order of magnitude. In the absence of the fundamental theory that should explain this "cosmic coincidence" [1] as well as clustering and other properties of those mysterious entities, numerous ideas have been suggested. In the quintessence scenarios of an accelerating expansion for the present day universe a promising approach was developed in the variable mass particles models [2], [3]. Such modifications of the particle physics theory have a number of problems (see for example [3]). One of the most fundamental problems is the following: although there are some justifications for coupling of the quintessence scalar ϕ to dark matter in the effective Lagrangian, it is not clear why similar coupling to the baryon matter is absent or essentially suppressed (such coupling would be the origin of a long range scalar force [4] because of the very small mass of ϕ). This "fifth-force" problem might be solved [5] if there would be a shift symmetry $\phi \rightarrow \phi + \text{const.}$ But the quintessence potential itself does not possess this symmetry although at present epoch it must be very flat [6].

Specific properties of neutrinos served as a basis for number of models [7], [8] concerning a possible relation of neutrinos with the dark energy sector. Recently a new idea has been suggested [9] that coupling of the neutrinos to a scalar allows to formulate the effective picture where the dark energy density depends on the neutrino mass (treated as a dynamical field). Such hypothesis is able to provide $w \approx -1$ only if the neutrino mass depends on the density of the background nonrelativistic neutrinos and the energy density in neutrinos is small compared to the energy density in the total dark energy sector. This cosmological scenario is very different from the quintessential one.

Here we present a theory where regular fermion matter and dark fermion matter can appear dynamically as different states of the same "primordial" fermion fields - the effect depending on the fermion energy density. Besides, nonrelativistic neutrinos can suffer a dynamical conversion into an effective dark energy leading to a transition into the state with the lower total energy density of the dark sector. Demonstration of this *new field theory effect* and exploration of some of its cosmological consequences are the main purposes of this letter. This and other dynamical effects appear in the framework of an alternative gravity and matter fields theory [10]- [15], Two Measures Theory (TMT). We will see that TMT is practically undistinguishable from General Relativity (GR) when fermion energy densities are of the order of magnitude typical for regular particle physics.

However before doing this in a systematic way in TMT we would like to give some filling of the origin of this new field theory effect on the basis of a somewhat extravagant model but in the framework of the standard field theory. In this context, we want to demonstrate that in some cases the energy-momentum tensor may contain the non-canonical fermionic term $\propto g_{\mu\nu}\bar{\Psi}\Psi$ that causes a negative fermion contribution to the pressure. In GR the fermion part of the action has usually the general form $\int L_f \sqrt{-g} d^4x$. Then a potential cosmological-like term $\propto g_{\mu\nu}L_f$ naively appears but the Dirac equation forces L_f to vanish. If however the action includes a topological density (for example we consider here $\Omega \equiv \epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}$ like in gauge theory) then the term $g_{\mu\nu}\bar{\Psi}\Psi$ can appear in the energy-momentum tensor. For illustration let us consider a model where in addition to the usual free fermion Ψ and the free massive scalar ϕ terms, the action contains also terms of the ϕ -to-fermion and ϕ -to-photon (parity

violating) couplings¹ $\int \lambda_f \phi \bar{\Psi} \Psi \sqrt{-g} d^4x + \int \lambda_{top} \phi \Omega d^4x$. By integrating out the ϕ field one can get (assuming that ϕ is slowly varying) the following fermion part of the effective action $S_{eff}^{(ferm)} = \int L_f^{(0)} \sqrt{-g} d^4x + \int \frac{\lambda_f \lambda_{top}}{m_\phi^2} \Omega \bar{\Psi} \Psi d^4x$, where $L_f^{(0)}$ is the Lagrangian density for the free fermion and m_ϕ is the mass of ϕ . Now the Dirac equation yields $\sqrt{-g} L_f^0 + \frac{\lambda_f \lambda_{top}}{m_\phi^2} \Omega \bar{\Psi} \Psi = 0$ that results in the appearance of the above-mentioned nonzero fermion contribution to the energy-momentum tensor $\propto \frac{\Omega}{\sqrt{-g}} g_{\mu\nu} \bar{\Psi} \Psi$.

Main ideas of Two Measures Theory. TMT is a generally coordinate invariant theory [10]- [15] with the action of the general form

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x \quad (1)$$

including two Lagrangians L_1 and L_2 and two measures of integration: the usual one $\sqrt{-g}$ and the new one Φ . The latter is built of four scalar fields φ_a ($a = 1, 2, 3, 4$)

$$\Phi = \epsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d. \quad (2)$$

Note that the measure Φ is a scalar density and a total derivative. To provide parity conservation, one can choose for example one of φ_a 's to be pseudoscalar. There are only two basic assumptions: (1) L_1, L_2 are independent of the measure fields φ_a (in this case the symmetry $\varphi_a \rightarrow \varphi_a + f_a(L_1)$ holds [11] up to a total derivative where $f_a(L_1)$ are arbitrary functions of L_1); (2) We proceed in the first order formalism where all fields, including vierbeins $e_{a\mu}$, spin-connection ω_μ^{ab} and the measure fields φ_a are independent dynamical variables. All the relations between them follow from equations of motion. It turns out that the measure fields φ_a affect the theory only via the scalar field

$$\zeta \equiv \Phi / \sqrt{-g} \quad (3)$$

which is determined by an algebraic constraint. The latter is exactly a consistency condition of equations of motion and *it determines ζ in terms of fermion and scalar fields*. After transformation to new variables (conformal Einstein frame), the gravity and all matter fields equations of motion take canonical GR form. All the novelty consists in the structure of the scalar fields effective potential, masses of fermions and their interactions to scalar fields as well as the structure of fermion contributions to the energy-momentum tensor: all these now depend on the fermion energy densities via ζ .

Scale invariant model. Under certain conditions TMT allows to realize GR and spontaneously broken non-Abelian gauge models [14] of particle physics. However, these aspects have no direct relation to the effects studied in the present letter. Therefore to simplify the presentation of the main results we will study a simplified model which is Abelian, does not include the Higgs field and quarks and chiral properties of fermions are ignored. In TMT there is no need [13], [14] to postulate the existence of three species for each type of fermions (like three neutrinos, three charged leptons, etc.) but rather this is achieved as a dynamical effect of TMT in normal particle physics conditions. The matter content of our model includes the dilaton scalar field ϕ , two so-called primordial fermion fields (the neutral primordial lepton N and the charged primordial lepton E) and electromagnetic field A_μ . The latter is included in order to show that the gauge fields dynamics in this model is canonical. Generalization to non-Abelian gauge models including also Higgs fields and quarks is straightforward [14].

The presence of the dilaton field ϕ allows to realize a spontaneously broken global scale invariance [12] which includes the shift transformation of ϕ . We will see that ϕ contributes to dark energy as a quintessence-like scalar field, and the shift symmetry is important [5] for resolution of the fifth-force problem [13]- [15].

We allow in both L_1 and L_2 all the usual contributions considered in standard field theory models in curved space-time. Keeping the general structure (1), it is convenient to represent the action in the following form:

$$\begin{aligned} S = & \int d^4x e^{\alpha\phi/M_p} (\Phi + b\sqrt{-g}) \left[-\frac{1}{\kappa} R(\omega, e) + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right] - \int d^4x e^{2\alpha\phi/M_p} [\Phi V_1 + \sqrt{-g} V_2] \\ & + \int d^4x e^{\alpha\phi/M_p} (\Phi + k\sqrt{-g}) \frac{i}{2} \sum_i \bar{\Psi}_i \left(\gamma^a e_a^\mu \bar{\nabla}_\mu^{(i)} - \bar{\nabla}_\mu^{(i)} \gamma^a e_a^\mu \right) \Psi_i \\ & - \int d^4x e^{\frac{3}{2}\alpha\phi/M_p} [(\Phi + h_N \sqrt{-g}) \mu_N \bar{N} N + (\Phi + h_E \sqrt{-g}) \mu_E \bar{E} E] - \int d^4x \sqrt{-g} \frac{1}{4} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} \end{aligned} \quad (4)$$

¹The coupling of ϕ to $\epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$ has been considered in Ref. [5] with the aim to realize the symmetry $\phi \rightarrow \phi + const$.

where Ψ_i ($i = N, E$) is the general notation for the primordial fermion fields N and E , $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, μ_N and μ_E are the mass parameters, $\vec{\nabla}_\mu^{(N)} = \vec{\partial}_\mu + \frac{1}{2}\omega_\mu^{cd}\sigma_{cd}$, $\vec{\nabla}_\mu^{(E)} = \vec{\partial}_\mu + \frac{1}{2}\omega_\mu^{cd}\sigma_{cd} + ieA_\mu$; $R(\omega, e) = e^{a\mu}e^{b\nu}R_{\mu\nu ab}(\omega)$ is the scalar curvature, e_a^μ and ω_μ^{ab} are the vierbein and spin-connection; $g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab}$ and $R_{\mu\nu ab}(\omega) = \partial_\mu \omega_{\nu ab} + \omega_{\mu a}^c \omega_{\nu cb} - (\mu \leftrightarrow \nu)$. V_1 and V_2 are constants of the dimensionality $(mass)^4$. When Higgs field is included into the model then V_1 and V_2 turn into functions (prepotentials) of the Higgs field. As we will see later, in the Einstein frame, V_1 , V_2 and $e^{\alpha\phi/M_p}$ enter in the effective potential of the scalar sector. Constants b, k, h_N, h_E are non specified dimensionless real parameters of the model and we will only assume that they *have closed orders of magnitude*; α is a real positive parameter.

The action (4) is invariant under the global scale transformations

$$\begin{aligned} e_\mu^a &\rightarrow e^{\theta/2} e_\mu^a, & \omega_{ab}^\mu &\rightarrow \omega_{ab}^\mu, & \varphi_a &\rightarrow \lambda_a \varphi_a & \text{where } \Pi \lambda_a = e^{2\theta} \\ A_\alpha &\rightarrow A_\alpha, & \phi &\rightarrow \phi - \frac{M_p}{\alpha} \theta, & \Psi_i &\rightarrow e^{-\theta/4} \Psi_i, & \bar{\Psi}_i &\rightarrow e^{-\theta/4} \bar{\Psi}_i. \end{aligned} \quad (5)$$

Except for a few special choices providing positivity of the energy in the Einstein frame, Eq.(4) describes *the most general TMT action* satisfying the formulated symmetries.

Varying the measure fields φ_a and assuming $\Phi \neq 0$, we get equations that yield

$$L_1 = sM^4 = \text{const} \quad (6)$$

where L_1 is now defined, according to Eq. (1), as the part of the integrand of the action (4) coupled to the measure Φ ; $s = \pm 1$ and M is a constant of integration with the dimension of mass. The appearance of a nonzero integration constant sM^4 spontaneously breaks the scale invariance (5).

All equations of motion resulting from (4) in the first order formalism contain terms proportional to $\partial_\mu \zeta$ that makes the space-time non-Riemannian and equations of motion - non canonical. However, in the new set of variables (ϕ and A_μ remain unchanged) which we call the Einstein frame,

$$\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p}(\zeta + b)g_{\mu\nu}, \quad \tilde{e}_{a\mu} = e^{\frac{1}{2}\alpha\phi/M_p}(\zeta + b)^{1/2}e_{a\mu}, \quad \Psi'_i = e^{-\frac{1}{4}\alpha\phi/M_p} \frac{(\zeta + k)^{1/2}}{(\zeta + b)^{3/4}} \Psi_i, \quad i = N, E, \quad (7)$$

the gravitational equations take the form

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{\kappa}{2} T_{\mu\nu}^{eff} \quad (8)$$

where $G_{\mu\nu}(\tilde{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian space-time with the metric $\tilde{g}_{\mu\nu}$ and

$$T_{\mu\nu}^{eff} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + \tilde{g}_{\mu\nu}V_{eff}(\phi; \zeta) + T_{\mu\nu}^{(em)} + T_{\mu\nu}^{(ferm, can)} + T_{\mu\nu}^{(ferm, noncan)}; \quad (9)$$

$$V_{eff}(\phi; \zeta) = \frac{b [M^4 e^{-2\alpha\phi/M_p} + V_1(v)] - V_2(v)}{(\zeta + b)^2}; \quad (10)$$

$T_{\mu\nu}^{(em)}$ is the canonical energy momentum tensor for the electromagnetic field; $T_{\mu\nu}^{(ferm, can)}$ is the canonical energy momentum tensor for (primordial) fermions N' and E' in curved space-time (including also interaction of E' with A_μ); $T_{\mu\nu}^{(ferm, noncan)}$ is the *non-canonical* contribution of the fermions into the energy momentum tensor

$$T_{\mu\nu}^{(ferm, noncan)} = -\tilde{g}_{\mu\nu}\Lambda_{dyn}^{(ferm)}, \quad \text{where} \quad \Lambda_{dyn}^{(ferm)} \equiv Z_N(\zeta)m_N(\zeta)\overline{N'}N' + Z_E(\zeta)m_E(\zeta)\overline{E'}E' \quad (11)$$

where $Z_i(\zeta)$ and $m_i(\zeta)$ ($i = N', E'$) are respectively

$$Z_i(\zeta) \equiv \frac{(\zeta - \zeta_1^{(i)})(\zeta - \zeta_2^{(i)})}{2(\zeta + k)(\zeta + h_i)}, \quad \zeta_{1,2}^{(i)} = \frac{1}{2} \left[k - 3h_i \pm \sqrt{(k - 3h_i)^2 + 8b(k - h_i) - 4kh_i} \right], \quad (12)$$

$$m_i(\zeta) = \frac{\mu_i(\zeta + h_i)}{(\zeta + k)(\zeta + b)^{1/2}}. \quad (13)$$

The origin of the mechanism generating $T_{\mu\nu}^{(ferm,noncan)}$ is somewhat similar to that discussed in the simple example at the end of Introduction, however here there is no need to integrate out the scalar ϕ . Note that $T_{\mu\nu}^{(ferm,noncan)}$ has the transformation properties of a cosmological constant term but it is proportional to fermion densities $\bar{\Psi}'_i \Psi'_i$ ($i = N', E'$). This is why we will refer to it as "dynamical fermionic Λ term". This fact is displayed explicitly in Eq.(11) by defining $\Lambda_{dyn}^{(ferm)}$. As we will see, $\Lambda_{dyn}^{(ferm)}$ becomes negligible in gravitational experiments with observable matter. However it may be very important for some astrophysics and cosmology problems.

The dilaton ϕ field equation in the new variables reads

$$\square\phi - \frac{\alpha}{M_p(\zeta + b)} \left[M^4 e^{-2\alpha\phi/M_p} - \frac{(\zeta - b)V_1 + 2V_2}{\zeta + b} \right] = -\frac{\alpha}{M_p} \Lambda_{dyn}^{(ferm)}, \quad (14)$$

where $\square\phi = (-\tilde{g})^{-1/2} \partial_\mu (\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \phi)$.

Equations for the primordial fermions in the new variables take the standard form where the standard electromagnetic interaction of E' presents also. All the novelty consists of the form of the ζ depending "masses" $m_i(\zeta)$, ($i = N', E'$) of the primordial fermions given by Eq.(13). The electromagnetic field equations are canonical.

The scalar field ζ is determined as the function of the ϕ and $\bar{\Psi}'_i \Psi'_i$ ($i = N', E'$) by the following constraint

$$\frac{1}{(\zeta + b)^2} \left\{ (b - \zeta) \left[M^4 e^{-2\alpha\phi/M_p} + V_1(v) \right] - 2V_2(v) \right\} = \Lambda_{dyn}^{(ferm)} \quad (15)$$

which is nothing but the consistency condition of equations of motion. Generically, the constraint (15) determines ζ as a very complicated function of ϕ , $\bar{N}'N'$ and $\bar{E}'E'$. Substituting the appropriate solution for ζ into the equations of motion one can conclude that in general, there is no sense, for example, to regard $V_{eff}(\phi; \zeta)$, Eq.(10), as the effective potential for the scalar field ϕ because it depends in a very nontrivial way on $\bar{N}'N'$ and $\bar{E}'E'$ as well. For the same reason, the $\Lambda_{dyn}^{(ferm)}$ term describes in general self-interactions of the primordial fermions depending also on the scalar field ϕ . Therefore it is impossible, in general, to separate the terms of $T_{\mu\nu}$ describing the scalar field ϕ effective potential from the fermion contributions. Such mixing of the scalar field ϕ associated with dark energy, on the one hand, and fermionic matter, on the other hand, gives rise to a rather complicated system of equations when trying to apply the theory to general situations that could appear in astrophysics and cosmology. Notice that in such a case, quantization of fermion fields may be problematic: inserting solution for ζ into the effective fermion "mass", Eq.(13), it is easy to see that the "free" primordial fermion equation appears to be very nonlinear in general. Considerable simplification of the situation occurs if for some reasons ζ appears to be a constant or almost constant. Fortunately this is exactly what happens in physically interesting situations.

Dark energy in the absence of massive fermions. In the fermion vacuum the constraint determines ζ as the function of ϕ alone:

$$\zeta = \zeta_0 \equiv b - \frac{2V_2}{V_1 + M^4 e^{-2\alpha\phi/M_p}}. \quad (16)$$

Then the effective potential of the scalar field ϕ results from Eq.(10)

$$V_{eff}^{(0)}(\phi) \equiv V_{eff}(\phi; \zeta_0)|_{\bar{\Psi}'\Psi'=0} = \frac{[V_1 + sM^4 e^{-2\alpha\phi/M_p}]^2}{4[b(V_1 + sM^4 e^{-2\alpha\phi/M_p}) - V_2]} \quad (17)$$

and the ϕ -equation (14) is reduced to $\square\phi + V_{eff}^{(0)'}(\phi) = 0$ where prime sets derivative with respect to ϕ .

The structure of the potential (17) allows to construct a model where zero vacuum energy is achieved without fine tuning [12] when $V_1 + sM^4 e^{-2\alpha\phi/M_p} = 0$. This allows to suggest a scenario where the "old" cosmological constant problem is solved.

In what follows we will assume $s = +1$ and $V_1 > 0$. Applying this as a model for dark energy in the FRW cosmology and assuming that the scalar field $\phi \rightarrow \infty$ as $t \rightarrow \infty$, we see that the evolution of the late time universe is governed by the sum of the cosmological constant

$$\Lambda^{(0)} = \frac{V_1^2}{4(bV_1 - V_2)} \quad (18)$$

and the quintessence-like scalar field with the potential

$$V_{q-l}^{(0)}(\phi) = \frac{(bV_1 - 2V_2)V_1 M^4 e^{-2\alpha\phi/M_p} + (bV_1 - V_2)M^8 e^{-4\alpha\phi/M_p}}{4(bV_1 - V_2)[b(V_1 + M^4 e^{-2\alpha\phi/M_p}) - V_2]}. \quad (19)$$

$\Lambda^{(0)}$ is positive provided $bV_1 > V_2$ that will be assumed in what follows. The needed smallness of $\Lambda^{(0)}$ can be reached either by the see-saw mechanism [12] playing with the ratio $V_1/|V_2|$ or choosing a large value of the parameter b which so far is a free parameter of the model. If $bV_1 < 2V_2$ then the potential $V_{eff}^{(0)}(\phi)$ has a minimum. $V_{eff}^{(0)}(\phi)$ decreases to $\Lambda^{(0)}$ monotonically if $bV_1(v_0) > 2V_2(v_0)$. In the model with $V_1 = V_2 = 0$, we get $\Lambda^{(0)} = 0$ and $V_{eff}^{(0)}(\phi)$ turns into the exponential potential of the quintessence field ϕ .

General Relativity and fermion families.

Reproducing Einstein equations. Analyzing Eqs.(8) and (9) it easy to see that they are reduced to the Einstein equations in the corresponding field theory model (i.e. when the scalar field, electromagnetic field and massive fermions are sources of gravity) if ζ is constant and $\Lambda_{dyn}^{(ferm)} = 0$ or at least

$$|T_{\mu\nu}^{(ferm,noncan)}| \ll |T_{\mu\nu}^{(ferm,can)}|. \quad (20)$$

According to Eqs.(11) and (12) this is possible if

$$Z_i(\zeta) \approx 0, \quad \Rightarrow \quad \zeta = \zeta_1^{(i)} \quad or \quad \zeta = \zeta_2^{(i)} \quad i = N, E, \quad (21)$$

where $\zeta_{1,2}^{(i)}$ are defined in Eqs.(12).

Fermion families birth effect in normal particle physics conditions. Let us now analyze some consequences of the constraint (15). Taking into account our assumption that b , k and h_i have close orders of magnitude, we see that $\zeta_{1,2}^{(i)}$ are of the order of magnitude close to that of b as well as to that of ζ_0 , Eq.(16). Then comparing $V_{eff}(\phi; \zeta_{1,2}^{(i)})$, $V_{eff}(\phi; \zeta_0)$, Eq.(17), and the l.h.s. of the constraint (15) we conclude that all of them have orders of magnitude close to that of the dark energy density (in the absence of fermions case). However, the r.h.s. of the constraint contains factors $m_i(\zeta)\bar{\Psi}'_i\Psi'_i$ ($i = N', E'$) which have typical orders of magnitude of the fermion canonical energy density $T_{00}^{(ferm,can)}$. Therefore it is evident that in normal particle physics conditions, that is when fermions are localized (in nuclei, atoms, etc.) and constitute the regular (visible) matter with energy density tens orders of magnitude larger than the vacuum energy density, the balance dictated by the constraint can be satisfied in the present day universe if the primordial fermions are in the states with values of ζ determined again by Eq.(21). Two constant solutions $\zeta_{1,2}^{(i)}$ ($i = N', E'$) correspond to two different states of the primordial leptons with *different constant masses* determined by Eq.(13) where we have to substitute $\zeta_{1,2}^{(i)}$ instead of ζ .

Similar to what we have done with primordial leptons N and E one can perform from the very beginning also with primordial quarks U and D . For this we need two additional mass parameters μ_U , μ_D and two additional dimensionless parameters h_U , h_D in the action. The appropriate values $\zeta_{1,2}^{(U)}$ and $\zeta_{1,2}^{(D)}$ might be defined then by equations similar to those in Eqs.(12). The need to describe a mixing of quarks requires more detailed discussion and solving a few technical problems that will be done in a separate paper. Ignoring here these questions we conclude that if the primordial fermion is in the normal particle physics conditions, then, according to the constraint, it can be either in the state with $\zeta = \zeta_1^{(i)}$ or in the state with $\zeta = \zeta_2^{(i)}$ ($i = N', E', U', D'$). Since the classical tests of GR deal with matter built of the fermions of the first generation (with a small touch of the second generation), one should identify the states of the primordial fermions obtained as $\zeta = \zeta_{1,2}^{(i)}$ with the first two generations of the regular fermions [13]- [15]. For example, if the free primordial electron is in the state with $\zeta = \zeta_1^{(E)}$ (or $\zeta = \zeta_2^{(E)}$), it is detected as the regular electron e (or muon μ) and similar for the electron and muon neutrinos with masses respectively:

$$m_{e(\mu)} = \frac{\mu_E(\zeta_{1(2)}^{(E)} + h_E)}{(\zeta_{1(2)}^{(E)} + k)(\zeta_{1(2)}^{(E)} + b)^{1/2}} \quad m_{\nu_e(\nu_\mu)} = \frac{\mu_N(\zeta_{1(2)}^{(N)} + h_N)}{(\zeta_{1(2)}^{(N)} + k)(\zeta_{1(2)}^{(N)} + b)^{1/2}}; \quad (22)$$

So, in the normal particle physics conditions, the scalar ζ plays the role of an additional degree of freedom determining different mass eigenstates of the primordial fermions identified with different fermion generations. One can show (this will be done in a separate publication) that the model allows to quantize the matter fields and provides right flavor properties of the electroweak interactions, at least for the first two lepton generations.

It turns out that besides the solution (21), there is only one more additional possibility to satisfy together the condition (20) and the constraint (15) when primordial fermion is in the normal particle physics conditions. This is the solution with $\zeta^{(i)} = \zeta_3^{(i)} \approx -b$ which we associate with the third generation of fermions. (for details see [13], [14]). The described effect of splitting of the primordial fermions into three generations in the normal particle physics conditions can be called "fermion families birth effect".

Resolution of the 5-th force problem. Fermion families birth effect (at the normal particle physics conditions) and reproducing Einstein equations (as the fermionic matter source of gravity built of the fermions of the first two generations) do not exhaust the remarkable features of the theory. Simultaneously with this the theory automatically provides an extremely strong suppression of the Yukawa coupling of the scalar field ϕ to the fermions observable in gravitational experiments. In fact, the Yukawa coupling "constant" is $\alpha \frac{m_i(\zeta)}{M_p} Z_i(\zeta)$ (see the r.h.s. of Eq.(14)) and for the mass eigenstates of the first two fermion generations it turns out to be zero automatically. This is the mechanism by means of which the model solves the long-range scalar force problem: in general, primordial fermions interact with quintessence-like scalar field ϕ , but this interaction practically disappears when primordial fermions are in the states of the regular fermions observed in gravitational experiments with visible matter.

Note that the fact that the same condition (21) provides simultaneously both reproduction of GR and the first two families birth effect seems very impressive because we did not make any special assumptions intended for obtaining this result.

Nonrelativistic neutrinos and dark energy. Due to the constraint (15), physics of primordial fermions at energy densities comparable with the dark (scalar sector) energy density turns out to be very different from what we know in normal particle physics. In this case, the non-canonical contribution $-\tilde{g}_{\mu\nu}\Lambda_{dyn}^{(ferm)}$, Eq.(11), of the primordial fermion into the energy-momentum tensor can be larger and even much larger than the canonical one. The theory predicts that in this regime the primordial fermion can not be in the states with ζ corresponding to regular fermion generations. Instead of this, for instance, in the FRW universe, the primordial fermion can participate in the expansion of the universe by means of changing its own parameters. We call this effect "Cosmo-Particle Phenomenon" and refer to such states as Cosmo-Low Energy Physics (CLEP) states [15].

As the first step in studying Cosmo-Particle Phenomena, here we restrict ourselves to the consideration of a simplified cosmological model where the spatially flat FRW universe is filled with a homogeneous scalar field ϕ and uniformly distributed *non-relativistic (primordial) neutrinos*. It is easy to show that in this case $\overline{N}'N' = \frac{const}{a^3}$ where $a = a(t)$ is the scale factor.

After averaging over typical cosmological scales (resulting in the Hubble law), the constraint (15) can be written in the form

$$(b - \zeta) \left[M^4 e^{-2\alpha\phi/M_p} + V_1 \right] - 2V_2 = \frac{(\zeta + b)^{3/2}}{(\zeta + k)^2} (\zeta - \zeta_1^{(N)}) (\zeta - \zeta_2^{(N)}) \frac{\mu_N n_0^{(N)}}{a^3}, \quad (23)$$

where the functions (12) and (13) have been used and $n_0^{(N)}$ is a constant determined by the total number of the cold neutrinos and antineutrinos. The l.h.s. of the constraint approaches a constant since we suppose a scenario where $\phi \rightarrow \infty$ as $a(t) \rightarrow \infty$. A possible solution of the constraint as $a(t) \rightarrow \infty$ is identical to the above studied case of the absence of massive fermions. There is however another solution where the decaying neutrino contribution $\mu_N n_0^{(N)}/a^3$ to the constraint is compensated by the appropriate behavior of the scalar field ζ . Namely if expansion of the universe is accompanied by approaching $\zeta \rightarrow -k$ then the r.h.s. of the constraint can approach the same constant as the l.h.s. does. This regime corresponds to a very unexpected state of the primordial neutrino. First, this state does not belong to any generation of the regular neutrinos. Second, the effective mass of the neutrino in this state increases like $(\zeta + k)^{-1}$ while the behavior of the $\Lambda_{dyn}^{(ferm)}$ term is $\Lambda_{dyn}^{(ferm)} \propto (\zeta + k)^{-2} u^\dagger u$. This means that at the late time universe, the canonical energy density of the non-relativistic neutrino $\rho_{(N)}^{(can)} \approx m_N(\zeta) \overline{N}'N'$ becomes much less than $\Lambda_{dyn}^{(ferm)}$. Third, such cold fermion matter possesses pressure and its equation of state in the late time universe approaches the form $p_{(N)} = -\rho_{(N)}$. Since $\Lambda_{dyn}^{(ferm)}$ approaches a constant we get $(\zeta + k) \propto a^{-3/2}$. A possible way to approach and get up a CLEP state might be spreading of the non-relativistic neutrino wave packet during its free motion (that may last a very long time).

We will assume that $V_1 > 0$, $bV_1 > 2V_2$ and $b > 0$, $k < 0$, $h_N < 0$, $h_N - k < 0$, $b + k < 0$.

Cosmological equations in the regime $\zeta \rightarrow -k$ read

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_p^2} [\rho_\phi + \rho_N] \quad (24)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{2\alpha k}{(b-k)^2 M_p} M^4 e^{-2\alpha\phi/M_p} + \mathcal{O}\left((\zeta + k)e^{-2\alpha\phi/M_p}\right) = 0, \quad (25)$$

where the scalar field ρ_ϕ and the CLEP state neutrinos ρ_N energy densities are respectively

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{bV_1 - V_2}{(b-k)^2} + \frac{b}{(b-k)^2}M^4e^{-2\alpha\phi/M_p}, \quad (26)$$

$$\rho_N = \mu_N \frac{(k - h_N)(b-k)^{1/2}}{(\zeta + k)^2} \frac{n_0^{(\nu)}}{a^3} = \frac{2V_2 + |b+k|V_1}{(b-k)^2} + \frac{|b+k|}{(b-k)^2}M^4e^{-2\alpha\phi/M_p}, \quad (27)$$

We have ignored here corrections $\sim \mathcal{O}(\zeta + k)$. The last expression in (27) results after using the constraint (23) and it allows a phenomenological description of the gas of the CLEP state neutrinos in terms of the scalar field ϕ . In the same approximation we get for the pressure of the gas of the CLEP state neutrinos $P_N \rightarrow -\rho_N$ as $a(t) \rightarrow \infty$ which means that the gas of the CLEP state neutrinos behaves as a sort of the dark energy. The total energy density and the total pressure in the framework of our toy model read

$$\rho_{dark}^{(tot)} \equiv \rho_\phi + \rho_N = \frac{1}{2}\dot{\phi}^2 + U_{dark}^{(tot)}(\phi); \quad P_{dark}^{(tot)} \equiv P_\phi + P_N = \frac{1}{2}\dot{\phi}^2 - U_{dark}^{(tot)}(\phi), \quad (28)$$

where the potential $U_{dark}^{(tot)}(\phi)$ of the effective dark energy sector is the sum

$$U_{dark}^{(tot)}(\phi) \equiv \Lambda + V_{q-l}(\phi), \quad \text{where} \quad \Lambda = \frac{V_2 + |k|V_1}{(b-k)^2}, \quad V_{q-l}(\phi) = \frac{|k|}{(b-k)^2}M^4e^{-2\alpha\phi/M_p}. \quad (29)$$

This means that the evolution of the late time universe in the state with $\zeta \approx -k$ proceeds as it would be in the standard field theory model (non-TMT) including *both the cosmological constant Λ and the quintessence-like field ϕ with an exponential potential.*²

Eqs.(17)-(19) and (29)) yield the remarkable result that

$$V_{eff}^{(0)}(\phi) - U_{dark}^{(tot)}(\phi) \equiv \frac{\left[\frac{b+k}{2}(V_1 + M^4e^{-2\alpha\phi/M_p}) - V_2\right]^2}{4(b-k)^2[b(V_1 + M^4e^{-2\alpha\phi/M_p}) - V_2]} > 0 \quad (30)$$

and in particular $\Lambda^{(0)} > \Lambda$. This inequality means that *the universe in "the CLEP state" has a lower energy density than the one in the "absence of fermions" case* and therefore there may be two different vacua: one is the usual vacuum free of the particles, which is actually a false vacuum, and the other, a true vacuum, incorporating neutrinos in CLEP state. This result does not imply at all that ρ_N is negative. One of the reasons of this effect consists in the reconstruction of $V_{eff}(\phi; \zeta)$, Eq.(10), when ζ , being determined by Eq.(16) in the absence of fermions case, becomes close to $-k$ in the CLEP state. In the transition to the CLEP state universe the crucial role belongs to the dynamics of cold neutrinos; the possibility of this transition has no relation neither to the values of V_1 and V_2 in the action (4), (which can be even equal zero, see footnote 2) nor to the value of the (positive) integration constant M^4 in Eq.(6). However the scale symmetry (5) and its spontaneous breaking by means of Eq.(6) have a decisive role in the structure of the *dynamically generating* effective potentials both in the absence of fermions case and in the CLEP state universe.

For a particular value $\alpha = \sqrt{3/8}$, the cosmological equations allow the following analytic solution for the late time universe ($\Lambda \neq 0$ is determined by Eq.(29)):

$$\phi(t) = \frac{M_p}{2\alpha}\varphi_0 + \frac{M_p}{2\alpha}\ln(M_pt), \quad a(t) \propto t^{1/3}e^{\lambda t}, \quad \text{where} \quad \lambda = \frac{1}{M_p}\sqrt{\frac{\Lambda}{3}}, \quad e^{-\varphi_0} = \frac{2(b-k)^2M_p^2}{\sqrt{3}|k|M^4}\sqrt{\Lambda}. \quad (31)$$

The mass of the neutrino in such CLEP state increases exponentially in time: $m_N|_{CLEP} \sim (\zeta + k)^{-1} \sim a^{3/2}(t) \sim t^{1/2}e^{\frac{3}{2}\lambda t} \sim \exp\left[\frac{3\lambda e^{-\varphi_0}}{2M_p}\exp\left(\frac{2\alpha}{M_p}\phi\right)\right]$.

Properties of the cosmological CLEP state solution allow to expect that spherically symmetric solutions in the regime close to the CLEP states may play an important role in the resolution of the halos dark matter puzzle. Note also that the constraint (15) allows many other so far unknown forms of fermion matter which deserve a special study.

Acknowledgements We thank S. Ansoldi, J. Bekenstein, S. del Campo, P. Frampton, K. Freese, P. Gondolo, G. Huey, P.Q. Hung, A. Kheifets, J. Morris, A. Nelson, H. Nielsen, Y. Jack Ng, E. Nissimov, S. Pacheva, L. Parker, T.

²Note that by tuning the parameters such that $V_2 + |k|V_1 = 0$ one can get $\Lambda = 0$ and then ϕ becomes the regular quintessence field with an exponential potential. Similar result for the CLEP state is achieved also in the model with $V_1 = V_2 = 0$.

Piran, R. Rosenfeld, E. Spallucci, A. Vilenkin and I. Waga for helpful conversations. One of us (E.G.) wants to thank the University of Trieste and the Michigan Center for Theoretical Physics for hospitality.

- [1] I. Zlatev, L. Wang and P. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999), astro-ph/9807002.
- [2] See for example: J.A. Casas, J. Garcia-Bellido, M. Quiros, *Class. Quant. Grav.* **9**, 1371 (1992); G.W. Anderson and S.M. Carroll, astro-ph/9711288; L. Amendola, *Phys. Rev.* **D62**, 043511 (2000); D.J. Holden and D. Wands, *ibid.* **D61**, 043506 (2000); A.P. Billyard and A.A. Coley, *ibid.* **D61**, 083503 (2000); N. Bartolo and M. Pietroni, *ibid.* **D61**, 023518 (2000); L.P. Chimento, A.S. Jakubi and D. Pavon, *ibid.* **D62**, 063508 (2000); D. Tocchini-Valentini and L. Amendola, *ibid.* **D65**, 063508 (2002); D. Comelli, M. Pietroni and A. Riotto, *Phys. Lett.* **B571**, 115 (2003).
- [3] G.R. Farrar and P.J.E. Peebles, *Astrophys.J.* **604**, 1 (2004), astro-ph/0307316; M.B. Hoffman, astro-ph/0307350; U. Franca and R. Rosenfeld, *Phys.Rev.* **D69**, 063517 (2004), astro-ph/0308149.
- [4] See for example: R.D. Peccei, J. Sola, C. Wetterich, *Phys.Lett.* **B195**, 183 (1987).
- [5] S.M. Carroll, *Phys. Rev. Lett.* **81**, 3067 (1998), astro-ph/9806099.
- [6] C. Kolda and D.H. Lyth, *Phys. Lett.* **B458** (1999), hep-ph/9811375.
- [7] P.Q. Hung, hep-ph/0010126; P.Q. Hung, H. Päs, astro-ph/0311131.
- [8] A. Singh, *Phys.Rev.* **D52**, 6700 (1995); X. Zhang, hep-ph/0410292; M. Blasone, A. Capolupo, S. Capozziello, S. Carloni, G. Vitiello, *Phys.Lett.* **A323**, 182 (2004), gr-qc/0402013.
- [9] R.Fardon, A.E. Nelson, N. Weiner, astro-ph/0309800; D.B. Kaplan, A.E. Nelson, N. Weiner, *Phys.Rev.Lett.* 93:091801,2004.
- [10] E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D53**, 7020 (1996); *ibid.* **D55**, 5970 (1997); *ibid.* **D56**, 3548 (1997), gr-qc/9702058; *ibid.* **D57**, 7200 (1998), gr-qc/9709059.
- [11] E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D60**, 065004 (1999), gr-qc/9905029.
- [12] E.I. Guendelman, *Mod. Phys. Lett.* **A14**, 1043 (1999), gr-qc/9901017; *ibid.* **A14**, 1397 (1999), hep-th/0106084; *Class. Quant. Grav.* **17**, 361 (2000), gr-qc/9906025; *Found. Phys.* **31**, 1019 (2001), hep-th/0011049.
- [13] E.I. Guendelman and A.B. Kaganovich, *Int. J. Mod. Phys.* **A17**, 417 (2002), hep-th/0110040.
- [14] E.I. Guendelman and A.B. Kaganovich, *Mod. Phys. Lett.* **A17**, 1227 (2002), hep-th/0110221.
- [15] E.I. Guendelman and A.B. Kaganovich, gr-qc/0312006; hep-th/0404099.